Designs of a miniaturized sapphire-loaded cavity for space borne hydrogen masers

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***Abstract*—The previous compact hydrogen maser with sapphire microwave cavity in the Beijing Institute of Radio Metrology and Measurement is not suitable for a space application in navigation systems since a bit large volume and weight. In order to reduce the total size of hydrogen maser atomic clocks further, in this work, the authors have performed the detailed analysis and some theoretical and experimental result in optimizing parameters in a TE011 mode sapphire-loaded cavity resonator more suitable for space borne hydrogen maser. The minimization of the total cavity volume and maximization of the quality factor are both sought finally. Methods of theoretical calculations, finite element simulation, and related experiments have been performed in the process of designing a sapphire loaded cavity. Based on the analysis, a miniaturized sapphire microwave cavity with the total volume of 3.04 dm3, the quality factor 67500, and frequency-temperature coefficient -59.7 kHz/℃ is developed. The experimental results are completely consistent with modeled values. In addition, the theoretical calculation result shows that the product of the z-component of the magnetic energy filling factor in the bulb region and the cavity TE_{011} mode Q-factor is 4.08E4 at 50 ℃ in the designed miniaturized sapphire cavity. In addition, two sapphire loaded cavities in the NICT, Japan and in IRCOM, France are detailedly compared with that of our designs.**

I. INTRODUCTION

The hydrogen maser atomic clocks were operated on the hyperfine splitting of the ground state of hydrogen atoms at about 1.42 GHz or 21 cm. The standing wave is exposed through the bulb region inside the resonant TE_{011} microwave cavity of the hydrogen maser that contains the very low density hydrogen atoms. This excited way of hydrogen atoms and microwave field has a distinct advantage that can remove the frequency shifts by first-order Doppler effects. Excellent performance in short-term frequency stability of the hydrogen maser atomic clocks is unsurpassed by other classical frequency standards [1]. An important application of the hydrogen maser is to provide a timing reference for spaceborne navigational satellites. Therefore, a compact and portable hydrogen maser would be of utmost importance for practical system considerations in space [2].

Because the bulk of microwave cavity occupies the almost three fourths total volume of the hydrogen atomic clocks, the reduction in the size and weight of the microwave cavity is extremely useful in the hydrogen maser for space application. In fact, these parameters are not very critical on earth. A sapphire-loaded microwave cavity can effectively reduce dimension of the hydrogen maser because the high permittivity of the sapphire (compared to vacuum) causes much better energy confinement. In addition, the lowest loss tangent of sapphire at microwave frequencies among any known dielectric materials is very beneficial to use in a miniaturized microwave cavity [3]. That is, a simple sapphire tube inside a cylindrical microwave cavity is the most compact design and can distinctly decrease the total size and weight of hydrogen atomic clock, furthermore without degrading excellent stability of hydrogen maser.

Ref. [4, 5] had some beneficial attempts about sapphire microwave cavity in the Beijing Institute of Radio Metrology and Measurement (BIRMM, China). The design was a titanium metal cylindrical cavity enclosing a sapphire tube which constitutes the gas bulb region of hydrogen atoms. The related size of microwave cavity has been shown in Table 1. The final total volume and weight of the atomic clock are 0.12 m^3 and about 55 kg, which have had an obvious decrease compared to that of classical hydrogen maser atomic clocks [6]. Unfortunately, this BIRMM sapphire hydrogen maser atomic clock is not

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suitable for a space application since its slightly large volume and weight.

In this work, we have performed the detailed analysis of TE_{011} mode sapphire-loaded cavity for more suitable aerospace demands. The criteria sought of our motive were both the minimization of the total volume of the cavity and the maximization of the quality factor at full steam. In addition, two sapphire loaded cavities in the Japanese National Institute of Information and Communications Technology (NICT, Japan) [7,8] and in the Microwave and Optical Communication Research Institute at the University of Limoges (IRCOM, France) [9,10] are detailedly compared with that of our designs.

Table 1 Comparison of some results in dimensions of volume and parameters of sapphire cavity for compact hydrogen masers in three Institutes. (Length unit: mm; Volume unit: mm^3)

	$2a$	$2b$	$2c$	h	d	V	Q	$\eta \cdot Q$
BIRMM [7,8]	176	95.1	82.6	175	6.25	4257483	51855	$\sim 3.4\times10^4$
NICT [10,11]	161.9	87.42	71.82	161.9	7.8	3332960	53380	$(3.18\sim3.36)\times10^4$
IRCOM, [12,13]	160	62	38	389.3	12	7827110	506000	

II. THEORETICAL ANALYSIS OF A SAPPHIRE CAVITY

The hydrogen maser operates on the hyperfine transition of (F = 1, m_F = 0) and (F = 0, m_F = 0) in the ground state of hydrogen atom. So, the intrinsic resonant frequency of TE_{011} microwave cavity must be close to this excited frequency of 1,420.4 MHz. In sapphire hydrogen maser, the hyperfine excitation of the hydrogen atoms is achieved in the sapphire bulb region locating in the axial center of the TE_{011} microwave cavity (as shown in Figure 1), in where the following characteristic equation of TE_{011} mode can be obtained from the Maxwell's equation [5]

$$\frac{\gamma_2\left[A_2J_0(\gamma_2 b)+B_2N_0(\gamma_2 b)\right]}{\left[A_2J_1(\gamma_2 b)+B_2N_1(\gamma_2 b)\right]}=\frac{\gamma_0\left[A_3J_0(\gamma_0 b)+B_3N_0(\gamma_0 b)\right]}{\left[A_3J_1(\gamma_0 b)+B_3N_1(\gamma_0 b)\right]}, \quad (1)$$

where

$$\begin{cases} A_2=(\gamma_2/\gamma_0)J_1(\gamma_0 c)N_0(\gamma_2 c)-J_0(\gamma_0 c)N_1(\gamma_2 c),\\ B_2=J_0(\gamma_0 c)J_1(\gamma_2 c)-(\gamma_2/\gamma_0)J_0(\gamma_2 c)J_1(\gamma_0 c),\\ A_3=-\dfrac{\pi}{2}\gamma_0 a N_1(\gamma_0 a),\\ B_3=\dfrac{\pi}{2}\gamma_0 a J_1(\gamma_0 a), \end{cases} \quad (2)$$

$$\gamma_i^2=\omega^2\mu_0\varepsilon_0\varepsilon_i-\pi^2/h^2,\ \ (i=0,\ 1,\ 2). \quad (3)$$

Here, J_0 and J_1 denote the Bessel function of the first kind of order zero and one; N_0 and N_1 denote the Bessel function of the second kind of order zero and one, respectively. By solving characteristic eq. (1), the distribution and density of magnetic and electric fields in the TE_{011} mode of sapphire cavity can be obtained.

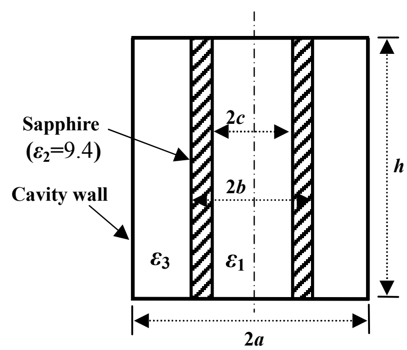


Figure 1 Configuration of TE_{011} mode of the sapphire microwave cavity. The inner diameter and inside height of metal microwave cavity are represented by "2*a*"and "*h*". "2*b*" and "2*c*" are the outer and inner sapphire tube diameters, respectively. $\varepsilon_1=\varepsilon_3=\varepsilon_0=1$ is the vacuum relative permittivity, $\varepsilon_2=9.4$ is relative permittivity of the sapphire tube along cylindrical axis at room temperature).

According to the stored energy (E_c) in the metal cavity, the loss energy (E_s) and (E_W) in the sapphire and the conducting cavity wall, and the storage volume (V), the quality factor (Q) is given by the following equations [7]

$$Q=\omega E_c/\left(E_s+E_W\right), \quad (4)$$

$$E_{\mathrm{c}} = \frac{V\mu_0 a^2}{4}\left\{\left(\frac{k_0}{x}\right)^2 L_1(\rho_1\rho_2 x) + \left(\frac{\rho_1 k_1}{\rho_2 y}\right)^2 \left[L_2(y) - \rho_2^2 L_2(\rho_2 y)\right] + \left(\frac{k_0}{\rho_1\rho_2 x}\right)^2 \left[L_3(x) - \rho_1^2 L_3(\rho_1 x)\right]\right\}, \tag{5}$$

$$E_{\mathrm{s}} = \frac{V\mu_0 a^2 \omega \tan\delta}{4}\left(\frac{\rho_1 k_1}{\rho_2 y}\right)^2 \left[L_2(y) - \rho_2^2 L_2(\rho_2 y)\right], \tag{6}$$

$$E_{\mathrm{w}} = \frac{V\mu_0 \omega \delta_s H_{z3}(x)^2}{4a(\rho_1\rho_2)^2} + \frac{V\mu_0\omega\delta_s \pi^2 a^2}{2h^3}\left\{\frac{L_1(\rho_1\rho_2 x)}{x^2} + \left(\frac{\rho_1}{\rho_2 y}\right)^2 \left[L_2(y) - \rho_2^2 L_2(\rho_2 y)\right] + \frac{L_3(x) - \rho_1^2 L_3(\rho_1 x)}{(\rho_1\rho_2 x)^2}\right\}, \tag{7}$$

where

$$\begin{cases} L_j(\theta) = H_{zj}^2(\theta) + H_{rj}^2(\theta) \\ \quad + 2H_{zj}(\theta) H_{rj}(\theta)/\theta, \quad (j = 1,\ 2,\ 3), \\ \rho_1 = b/a,\ \rho_2 = c/b, \\ k_0 = \omega^2\mu_0\varepsilon_0,\ k_1 = \omega^2\mu_0\varepsilon_1, \\ x = a\left[k_0^2 - (\pi/h)^2\right],\ y = b\left[k_1^2 - (\pi/h)^2\right]. \end{cases} \tag{8}$$

Here, $\tan\delta$ is the dielectric loss of the sapphire, and δ_s is the skin depth of the cavity wall. H_{zj} and H_{rj} are the *z*-axis component and radial component of magnetic field in the region *j*, respectively.

III. RESULTS AND DISCUSSION

A. Theoretical calculation and FEM

In our sapphire cavity of the hydrogen maser for theoretical analysis, a low-loss TE_{011} mode solid resonator was used, as shown in Figure 1, which was made of aluminum metal because of its nonmagnetic characteristic and excellent low thermal expansion coefficient. A cylindrical sapphire tube is placed in the center of the microwave cavity. The crystallographic [0001] direction of single-crystal sapphire aligned to the cylinder axis of cavity. Some former experimental results [11,12] about anisotropic permittivity, and dielectric loss coefficients of sapphire are used in the process of calculation.

In Figure 2, calculations result in the length of diameter and height of sapphire microwave cavity dependence of the resonant frequency or quality factor of hydrogen maser according to eq. (1). The resonant frequencies go near to rise linearly with the decrease of the length of diameter and height, however, the quality factor increase with expanding the cavity dimension. No sharp peak or jump occurs in whole curves. When the radius "*a*" of cavity changes 1 mm, the resonant frequency alters about 4 MHz, correspondingly. For an example, when the length of cavity fixed in 175 mm, the resonant frequency is about 1.419 MHz and quality factor is 50600, which is consistent with the results of the previous designs [4].

According to the reported theoretical and experimental results [4,5], the average radius of sapphire tube equals to half of the radius of the microwave cavity showing the optimal characteristics. But, for the miniaturized sapphire cavity, both the inner and outer radius of sapphire tube must be repeatedly consider for reducing the cavity volume as full as possible [13]. Therefore, in the theoretical calculation, we will take into account the effect of both the radius and thickness of sapphire tube. Figure 3 only shows the calculative data about the thickness ($d = b - c$) of sapphire tube dependence of the resonant frequency and quality factor in the cavity. With broadening of the sapphire wall, both resonant frequency and quality factor decrease consistently. In our previous compact hydrogen maser [4], the thickness of sapphire tube is 6.25 mm, corresponding resonant frequency 1.433 MHz, which agrees very well with the experimental results. Due to the non-cylindrical sapphire cavity (adding two covers in the ends) and frequency-temperature effects in experimental microwave cavity, a bit larger frequency than that of hydrogen excitation must be required.

The previous sapphire microwave cavity in BIRMM is not suitable for a space borne H-maser since a bit large volume and weight. According to the trend of curves in Figure 2 and Figure 3, the resonant frequency remains unchanged as long as reducing the volume of the microwave cavity and adding the thickness of sapphire tube together. Because the hollow centre regime of sapphire bulb has another purpose in where the hydrogen atoms were excited by magnetic field, in order to remain the space and time enough, the inside ring radius of sapphire bulb cannot be contracted unlimitedly. We design the Al metal cylinder has a height *h* = 172 mm so as to sufficiently ensure the uniform region of magnetic field as that of BIRMM's previous design. According to the theoretical calculation from eq. (1), an inner diameter $2c$ = 38 mm and outer diameter $2b$ = 62 mm of the sapphire tube, an inner diameter $2a$ = 150 mm of microwave cavity, and a constant resonant frequency of 1.42 GHz can be obtained. In addition, the quality factor of this novel miniaturized sapphire-loaded cavity is 67500 calculated from eq. (4).

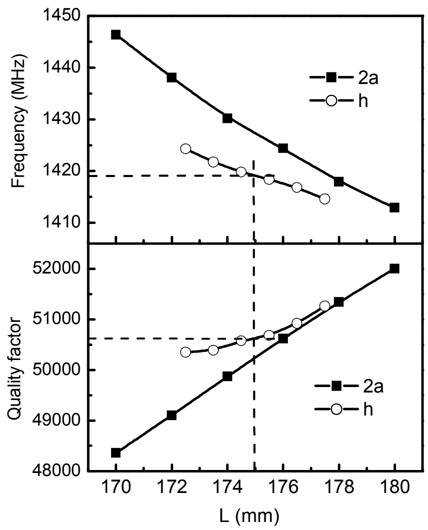


Figure 2 The length of diameter or height of microwave cavity dependence of resonant frequency and quality factor of the sapphire hydrogen maser.

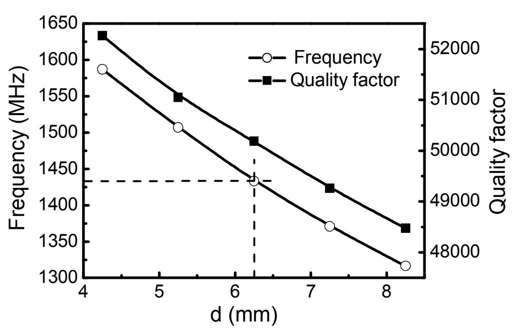


Figure 3 The thickness of sapphire tube dependence of cavity frequency and quality factor of the sapphire hydrogen maser.

These results meant a 28.6% smaller in volume of cavity and larger in quality factor than that of our previous design in BIRMM (as shown in Table 1). The outer diameter of the sapphire tube becomes 62 mm, which is same with ref. [10] results but smaller than that in the ref. [7] design. The cavity diameter and height become 150 mm and 172 mm, which are significantly different from Opiquet's design [10] but is some similar with the results of Morikawa et al's design [7,8]. The total volume of the microwave cavity is the smallest among the known sapphire cavity for hydrogen maser.

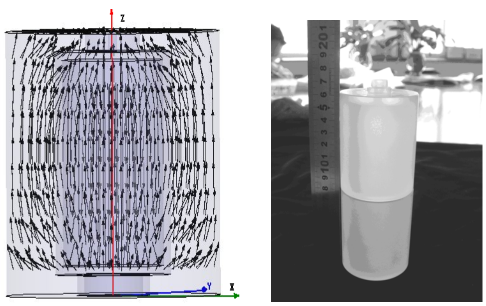
Because the hyperfine excitation of the hydrogen atoms is achieved in the bulb region by the driving field of a cylindrical TE_{011} mode cavity, the product of the z-component of the magnetic energy filling factor (η) in the bulb region and the cavity TE_{011} mode Q-factor ($\eta \cdot Q$) is another important parameter for the practical operation of the hydrogen maser atomic clocks. At 50℃, the theoretical calculation obtains $\eta \cdot Q = 4.08 \times 10^4$ in our designed miniaturized sapphire cavity, which is about 25% larger than that of Morikawa et al. [7].

In order to further test and calibrate these results of our theoretical calculations, the designed miniaturized sapphire cavity is simulated using finite element method (FEM). The sizes of cavity in the FEM are a = 75 mm, b = 31 mm, c = 19, and h = 172 mm, respectively. The simulated results about the magnetic pattern inside microwave cavity were shown in Figure 4. The TE_{011} mode of FEM results is well consistent with that of the theoretical analysis. In addition, the resonating frequency 1.4210 GHz and quality factor 67250 in our miniaturized sapphire cavity can be obtained, respectively, which are fully fit for the resonant qualification of hydrogen maser. Compared with the results obtained by Morikawa et al. [7,8], our new designs in miniaturized microwave cavity represent an obvious improvement regardless of the total volume or physical characteristics.

B. Experimental results and discussion

For the miniaturized sapphire cavity used in our experimental set-up for space borne hydrogen maser, the final design was made from a single-crystal sapphire tube enclosed in a cylindrical silver-plated-aluminum shield as shown in Figure 4 (A). And, Figure 4 (B) also shows the sapphire bulb inside the miniaturized microwave cavity. Two endsides of the sapphire tube was closed by same single-crystal sapphire sample with its c-axis aligned to the cylinder axis. The results show that the influence in cavity frequency and quality factor of this sapphire bulb configuration with two covers in the non-cylindrical shape can be ignored.

In order to approximate adjust the resonant frequency, the cavity height (172 mm) and sapphire sizes are precisely machined and inside radius of Al cavity is diminutively processed based on designs. When the inner radius of the miniaturized sapphire cavity is 75 mm, the experimental results in resonant frequency and quality factor is 1,425.115 MHz and 64485, respectively, i.e. theoretical calculation results, finite element simulations, and experimental data are found to be in very good agreement. In addition, it is worth indicating here that experiments identify the change of 4 MHz resonant frequency as altering 1 mm inner radius of the miniaturized cavity, which is also consistent with results of calculation and FEM.



(A) (B)

Figure 4 (A) The magnetic field energy distribution plot for the TE_{011} mode in the miniaturized sapphire cavity. Most of the magnetic field energy is confined in the central region of sapphire bulb. The arrows represent the direction of microwave magnetic field. (B) The sapphire bulb inside the miniaturized microwave cavity.

Because the actual microwave cavity in atomic clock is operated at about 50℃, the frequency-temperature coefficient in the new miniaturized cavity must be renewedly estimated. Our related experiment is undertaken in an airtight box which can offer the different temperature points. And, according to the cavity frequency at every temperature, the frequency-temperature coefficient near work point can be calculated. For an example, when choosing at two temperature points at 50℃ and 55℃, the cavity frequency can be measured as 1419.265 MHz and 1418.966 MHz, respectively. The frequency-temperature coefficient can be obtained as –59.7 kHz/℃, which is very close to that of the previous compact hydrogen maser (–58.8 kHz/℃) in BIRMM. The resonant frequency of cavity at 50℃ is also close to hyperfine frequency of hydrogen atoms. Although much thicker in sapphire wall of new microwave cavity, the total mass of sapphire does not relevantly increase. This also leads to the frequency-temperature coefficient unchangeable.

IV. CONCLUSION

In conclusion, based on the theoretical calculations, FEM simulation, and related experiments, a novel miniaturized TE_{011} mode sapphire cavity for spaceborne hydrogen maser is designed. In whole designed process, the theoretical analysis is agreed very well with measured characteristics. This miniaturized sapphire cavity has two main advantages over previous in BIRMM. On the one hand, due to the volume of microwave cavity becomes smaller, the total size and weight of hydrogen maser atomic clocks can be decreased obviously. Therefore, it is much fitter for a space application. On the other hand, the small diameter of single-crystal sapphire for the miniaturized cavity is easier to grow and expense cost is much lower to hydrogen maser atomic clocks.

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